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REASON AND COMMON SENSE

CRITICAL reason, especially the metaphysical brand of it, often runs counter to naïve reason, otherwise known as common sense. While this is no justification for rejecting the philosophical conclusion out of hand and absolutely, it is nevertheless a challenge to reexamine its grounds. Common sense has triumphed over some old scraps of philosophy, and it is rather likely to triumph over some that are not so old. It is time to sound a note of warning to philosophers not to be too reckless in flouting common sense.

Zeno thought he had proved that Achilles can not catch the tortoise; that, in fact, all motion is impossible; that all things are really at rest; that what we call motion is mere appearance. Common sense rebelled against such absurdities, and its verdict is now sustained by critical reason.[1]

Kant thought he had proved his antinomies, *i. e.*, that certain pairs of contradictories are both true. Critical reason now concurs with common sense in the conviction that, though Kant's theses and antitheses may in some fashion all be considered fairly demonstrable, it is only because they are to be understood in different senses. Contradictories can not both be true in the same sense.

Some vagaries of recent philosophizing stand a good chance to meet, at the hands of common sense and enlightened critical reason, the same fate that has befallen these older gems of metaphysical wisdom.

The starting post on the modern road to "astonish common sense" is the *point*. It has position only, but no extension. Hence we can pack millions of points in the tiniest space, like the devils dancing on the point of a needle in monkish philosophy. Points are tricky, too, like the imps aforesaid. Their unique quality of lacking extension, so that we can neither see them nor touch them nor hold them down to any sort of sensible ordinary behavior, makes them very slippery customers. We have to be careful how we talk about them. If one says that there are as many points in an inch as in a

[1] *Cf.* Spaulding, *The New Rationalism*, pp. 166–68. *Cf.* also Montague, *Studies in the History of Ideas*, pp. 245–48.

million miles, because the number is infinite in both cases, we can not conclusively deny it, though we may prefer the more cautious form, "there *may be* as many." The objection to the positive statement is that every positive assertion is a limitation, and an essential mark of the infinite is the absence of all limits. To say that the number is the same in both cases implies that we have "thought around" both infinites, encompassed them, bounded and determined them in thought so that we know they are equal. But whatever can be thus determined in thought is finite, not infinite. Hence it is inconsistent with the vague nature of infinity to say positively that there are as many points in an inch as in a million miles. "There may be as many," is not quite such a shock to common sense, and it is philosophically safer.

We can treat instants in the same way as points. An instant is a time-point. It has temporal position but no duration, just as a space-point has spatial position but no extension. We can therefore safely say that there may be as many instants in a minute as in a year, for the number is infinite in both cases.

But can we say that "there are just as many years in eternity as there are minutes"?[2] Years and minutes are in a very different category from instants; they are temporal units of measurement, and unlike instants *they have duration.* We can not juggle with them as we can with points and instants. Since there are some half million minutes in a year ($60 \times 24 \times 365\frac{1}{4} = 525{,}960$), common sense would say that in any period whatever, whether finite or infinite, the years must be multiplied by half a million or more in order to equal the minutes. Possibly in the long run common sense may get the better in this argument. The burden of proof in all fairness rests upon the bizarre assertion, and how is its author going about to prove it? If some great benefit were certain to accrue to mankind by the demonstration there would be more encouragement to undertake the task, but the profit to any mortal is far to seek. Lost souls might indeed be deluded with the hope of a shorter term if every minute counted off a whole year. Otherwise the utility of this time-scheme is imperceptible.

As for the method of proof there is of course that ingenious device much in vogue just now of setting up a one-one correspondence between two infinite series. In this case one series will be made up of years and the other of minutes.

[2] W. Curtis Swabey makes this assertion in his article, *Mr. Bradley's Negative Dialectic and Realism,* this JOURNAL, Vol. XVI., No. 15, p. 411. Bertrand Russell says that the *days* and years "in all time" are equal. (*Mysticism and Logic,* p. 91.)

1st y., 2d y., 3d y. ··· *ad infinitum.*
1st m., 2d m., 3d m. ··· *ad infinitum.*

As Bertrand Russell naïvely remarks, "There are obviously just as many numbers in the row below as in the row above, because there is one below for each one above."[3] It is true he was speaking of another case of one-one correspondence, but we have only to reverse "above" and "below" to adapt the remark to this case.

Now are the above infinites equal each to each, and both equal to the same eternity? Which eternity is meant, past, future, or the whole realm of Father Time, past, present, and future? Are these several eternities all equal, inasmuch as they are all infinite? Are all infinites equal? That point seems to be quietly assumed in setting up one-one correspondences running *ad infinitum.*

We are a bit suspicious of the soundness of this one-one proof. In the first place, asserting—or assuming—equality of infinites is inconsistent with the vague and indeterminate nature of infinity. But, in the second place, even if we ignore that objection, we have still to inquire in what respects infinites are equal. Those now under consideration are temporal infinites, year-and-minute infinites. Are they equal in actual duration or in the serial number of terms? The latter kind of equality is a necessary presupposition of the assertion that the years and minutes of eternity are equal in number. How then are they related in actual duration? The year-infinite looks as if it must be half a million times longer than the minute-infinite. Equality in actual duration of two infinites, the one made up of any number of years and the other of the same number of minutes, is a self-contradiction. But both must be equal to eternity —the same eternity too—to make good the assertion that the years and minutes of eternity are equal in number. And "equal to eternity" surely means equal in actual duration. Here then we have contradictory presuppositions of that bizarre assertion. Year-and-minute infinites must be equal in the serial number of terms, and they must also be equal in actual duration. The one-one proof is plump up against an *impasse.*

But some one may say, "infinites are infinites, just as pigs are pigs, and when you have run a series up to infinity there is nothing more to be said." Giving all due weight to that profound remark, still it is hardly to be supposed that any one would consciously and with grim determination maintain that all infinites are equal. That many do subconsciously assume it is evident. Let us see where that assumption would lead us. It would indeed be a fine scheme for proving anything we wish. Mill suggested that in some other world

3 *Mysticism and Logic,* p. 86.

—evidently a fancy-free sort of a world—$2+2$ may for aught we know equal 5. But by assuming equality of infinites we can prove that theorem for our own prosaic world.

$$\frac{2+2}{0} = \infty ; \frac{5}{0} = \infty .$$

If infinites are equal then

$$\frac{2+2}{0} = \frac{5}{0} \text{ and } 2+2 = 5.$$

Also we can prove that $1=2$.

$$\tfrac{1}{0} = \infty = \tfrac{2}{0}. \quad \therefore 1 = 2.$$

But instead of relying on an independent proof that the years and minutes of eternity are equal in number—a proof which limps painfully if indeed it gets on at all—suppose we simply fall back on our brand new definition of infinity. "The infinite is that which is in one-one correspondence with part of itself." The reasoning may possibly run somewhat in this fashion: Eternity is infinite, and *by definition* is in one-one correspondence with its years, also with its minutes; therefore the years are in one-one correspondence with the minutes; therefore the years and minutes are equal in number.

There is a double confusion of ideas in this kind of reasoning. Years and minutes, by reason of having duration as well as temporal position, can not be handled like instants which have no duration. The instants in any period, however brief, may be considered infinite, and that opens the door for one-one correspondences *ad libitum*. We are never brought up with a sharp turn by any shortage of instants. Not so with years and minutes. In a century-series the year-terms would be just 100 while the minute-terms would be $100 \times 525{,}960$. No possibility of one-one correspondence in that case. No amount of stretching will serve to make up for the shortage of years.

Another kind of confusion appears in the notion of one-one correspondence of eternity with either its years or its minutes. We must break up eternity into pieces of some sort in order to get a series representing the "whole." If the elements of eternity are taken to be instants, then years must be treated in the same way so that the "part" may be a "proper" part. We thus get a correspondence, but it is not the one we want—not eternity with years, but instants with instants. In fact there is no legitimate way to get the correspondence we want, either of eternity with its years or minutes, or years and minutes with each other.

In the case of some infinites the impossibility of a series representing the "whole" is obvious at once; the nature of the infinite in question distinctly resists and resents the conception of serial order. Divine compassion is infinite, but the idea of chopping it up into a series in order to set up one-one correspondence with part of itself, is ridiculous.

Of course we do not say, or imply, that any one has actually set forth in detail the line of reasoning sketched above for a possible application of the new definition of infinity; we are merely "supposing" a case. Yet it is highly probable that the new definition is in some vague way at the bottom of the conclusion that the years and minutes of eternity are equal.

Quite aside, moreover, from confusion of ideas in *applying* the new definition, there is another difficulty about this line of reasoning which is absolutely fatal to it. The new definition itself is based upon the same dubious assumptions and arbitrary forcing of serial relations in order to make out a specious appearance of one-one correspondence, which we have already found to involve absurd and impossible consequences. It is true that we may have a one-one correspondence of whole and part in the case of points and instants, but otherwise the correspondence fails, so that the definition is faulty in that there are many real infinites not in one-one correspondence of whole and part. This is conspicuously true of our next example of "astonishing common sense."

In the infinite series of whole numbers "there are *as many even* integers as there are *odd and even*."[4]

The alleged "proof":

$$1,\ 2,\ 3,\ 4,\ \cdots\ \textit{ad infinitum.}$$
$$2,\ 4,\ 6,\ 8,\ \cdots\ \textit{ad infinitum.}$$

If this one-one correspondence is a sound proof in the case of even integers, we can extend its range indefinitely. Squares, cubes,

[4] Spaulding, *The New Rationalism*, p. 160. The same assertion "proved" in the same way, *i. e.*, by one-one correspondence, is found in numerous recent books and articles. It is one of the hall-marks of being up to date. William James is more cautious than other authors in his statements about integers. "Thus, in spite of the fact that even numbers, prime numbers, and square numbers are much fewer and rarer than numbers in general, they appear to be equally copious for purposes of counting." (*Some Problems of Philosophy*, p. 175.) Plenty of even numbers "for purposes of counting" is a proposition very different from "just as many even numbers as odd and even." But James immediately goes on to make his statement stronger. "Since every integer, odd or even, can be doubled, it would seem that even numbers thus produced can not in the nature of things be less multitudinous than that series of both odd and even numbers of which the whole natural series consists." We shall presently have a "look in" to ascertain the legitimate effect of this "doubling" process.

all sorts of powers or multiples, may be in like manner set forth in one-one correspondence with the whole series of natural integers.

$$1,\ 2,\ 3,\ 4,\ \cdots \textit{ ad infinitum.}$$
$$1^n,\ 2^n,\ 3^n,\ 4^n,\ \cdots \textit{ ad infinitum.}$$

The original assertion is sufficiently "astonishing" to common sense, but with high values of the exponent n the "astonishment" becomes fairly overwhelming. The numerical magnitude of the integers in the bottom row speedily passes clear comprehension. What sort of mental grasp can we have of the billionth power of a million? Yet, if the one-one proof is sound, there are just as many of these billionth powers as there are integers in the whole series, billionth powers included.

Is the one-one proof sound? A clear issue is joined, common sense *vs.* critical reason. Common sense says that the even numbers are only half of the whole series of integers; current philosophy teaches that they are equal to the sum of the odd and even, a part equal to the whole.

What is the meaning of *ad infinitum*? Both of the above series may be conceived as being continued till their last terms are both infinite, or till the number of terms in both is infinite. The first meaning is just as pertinent as the second—nay more, it is the one legitimate meaning, as we shall see in the sequel. But what follows its acceptance? At any point of equal numerical magnitude of last terms in both series, *e. g.*, 8, there are twice as many odd-and-even integers as even integers.

$$1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8.$$
$$2,\ 4,\ 6,\ 8.$$

At the infinite point of equal last terms in both rows the same is true and common sense wins. It is only by ignoring the first meaning and assuming the second as if it were the only possible meaning of *ad infinitum,* that symbolists "prove" their case.

We can indeed, as James says, always double any term in the top row, and that *seems* to ensure the possibility of keeping the rows even. But *the double must also have its place in the upper row,* together with all lesser integers; for the top row must contain *all* integers. Thus it is continually running ahead of the lower row; its speed in the race to its infinite goal is just twice as great as that of the lower row if both series are developed in strict compliance with the conditions of the problem. No integer inserted in the bottom row can be omitted from the top row. But whenever we stop with the same number of terms in both rows—as symbolists always do in order to make a show of one-one correspondence—there will be terms

in the bottom row not found in the top row. Thus if we stop with six terms in each row, three of those in the lower row will be absent from the top row.

1, 2, 3, 4, 5, 6.
2, 4, 6, (8, 10, 12).

Of course if we thus leave out half of the odd-even series, *i. e.*, leave out 7, 8, 9, 10, 11, 12, it will be only equal to the even series, not its double. But instead of mutilating the odd-even series by leaving out a moiety of it, let us give it its just allowance of integers in full tale. When we balance 9 in the top row with 18 in the lower, we ought at once to write 18 in the top row also, and proceed to fill in 10, 11, 12, 13, 14, 15, 16, 17, in the gap between 9 and 18. The initial condition of the problem, *viz.*, the top row to contain *all* integers and the lower to contain only even integers, demands precisely that sort of genesis and development of the two series. This effectually *shuts out* the second meaning of *ad infinitum*, the sole basis, the absolute *sine qua non* of the anti-common-sense conclusion that the even integers equal the odd-and-even. Russell's naïve remark that "there are obviously just as many numbers in the row below as in the row above, because there is one below for each one above," is "obviously" true only when he, regardless of the plain conditions of the problem, has manipulated the series so that it must be true.

Here then is an actual infinite, the series of natural integers, which does not conform to the much-vaunted "new definition" of infinity. And this is by no means a solitary exception.

The new definition reverses, or attempts to reverse, the relation of finite and infinite. We used to think, and some of us still think, that the finite is the real standard of comparison, the known term of the couplet finite-infinite. It is determinate and positive in content; the infinite is the not-finite. Now the attempt is made to give the infinite a positive content and the finite is the not-infinite.[5] But after all is said and done it is the infinite that remains vague and indeterminate, so that definite assertions about infinites are risky. And it is precisely these risky assertions and dubious assumptions that underlie the conclusions adverse to common sense.

Passing beyond the rational determinate bounds of the finite is a plunge into the vague, the mysterious, the unknown. Infinity is not a standard entity of uniform extent or value, as it seems to be regarded in setting up one-one correspondence series running *ad infinitum*. It is indeed a broad mantle, but not quite adequate to cover all sins of omission or commission on the way to it or in the shadow

[5] "It then follows that a finite number is one that is not infinite." Spaulding, *The New Rationalism*, p. 453.

of its mystery. Some of the extravagant assertions about infinites seem to be mere mental pyrotechnics flashing out of murky depths like sky-rockets shot off in the dark. They need not be taken too seriously. Somewhat of that nature is the following: "From an infinite series any number of members can be taken, and to an infinite series any number can be added, without either increasing or diminishing it."[6] Now as regards the truth of this flash of inspiration—or *ignis fatuus,* whichever it may be—it is on all fours with the assumption that all infinites are equal. Once admit that this infinite may be greater than that, then it is not irrational to suppose that adding the difference to the less, or taking it from the greater, will in either case make them equal. We do not assert that result as a positive fact; we have to be careful how we talk about infinites. But the mere possibility of it subverts the *dictum* above cited. An instance of one infinite growing and another shrinking is right before our eyes—the eyes of the mind—all the time. Every added hour in the flight of time is an accretion to past eternity and a shrinkage of future eternity. That infinites can neither be increased nor diminished is, consequently, just one more risky assertion about infinites. At all events it has no such status of solid verity as to form a safe inferential basis for any other "astonishing" assertion about infinites. No use trying to prove by it that eternal years and minutes are equal, or that the even integers equal the odd-and-even. One risky assertion does not prove another of the same ilk.

The habit of indulging in sweeping assertions may usually be traced either to gross ignorance or pride of intellect. Sound scholarship tends to caution and moderation. Of all fields for exploiting a dogmatic temper the vague and boundless infinite would seem to be about the least appropriate. It might be thought alluring to some minds because of the difficulty of refuting their bizarre assertions about infinites. Such a consideration would be proper enough if it were always based upon solid conviction of the truth of their statements. But there may be a suspicion that they feel safe because they are operating in an elusive realm of vagueness and mystery. One of Russell's *dicta* tends to foster such a feeling of security—not intentionally of course. In the note on page 87 of *Mysticism and Logic,* he says: "Although some infinite numbers are greater than some others, it can not be proved that of any two infinite numbers one must be the greater." Hence, possibly, the complacency with which

6 May Sinclair, *A Defense of Idealism,* p. 226. This is apparently not a hit from the author's own bat. Though not a direct quotation it is obviously based upon some one of the inspired *dicta* of Cantor or Russell. The inference therefrom, "that a finite series is not, in any sense, part of an infinite series," hardly needs a "mathematician" to reject it.

symbolists assume equality of infinites in order to make out a plausible showing of one-one correspondence. But that complacency is not well grounded. This inspired *dictum* that inequality of infinites "can not be proved," is just another example of risky assertions about infinites. *Reductio ad absurdum* has classical sanction as a method of proof, and we have shown above that equality of infinites involves the absurd conclusion that $1=2$. Thus its contradictory, the possible inequality of infinites, is fairly established. "All infinites are equal" is logically refuted by "Some infinites are not equal."

It is only points and instants that admit of genuine one-one correspondence between part and whole. Their amazing capacity for this trick is wholly due to the fact that points have no extension and instants no duration. A fancied analogy between them and units of measurement, spatial or temporal, such as feet and inches, minutes and years, has misled philosophers into the fallacy of illegitimate extension of the notion of one-one correspondence. They have ignored the fact that when we pass from unextended points to extended units, we are in quite another universe of discourse. The illusion is partly due to confusion of ideas in the familiar example of the points in one inch and one foot. A line a foot long is made up of two kinds of parts, extensionless points and extended units of measurement, and it is all too easy to get these mixed up in thought. An inch is a "proper part" of a foot, *i. e.*, a part like the whole in that it is an extended line. But the points in an inch are not a "proper part" of a foot; the extensionless is not like the extended. Although the inch is a "proper part" of the foot, feet are not, and can not be, in genuine one-one correspondence with inches, nor years with minutes. But some vague notion of such correspondence underlies the illegitimate extension of the one-one proof from points and instants to units of measurement, and leads to such conclusions as that the years of eternity equal the minutes of eternity. Those impish points and instants have played a sly trick on your learned and dignified philosopher.

The inaccuracy of the new definition of infinity is a serious matter, for infinites and one-one correspondence cut a great figure in current philosophical discussion. Indeed, not alone in the latest philosophical creeds but all down through the ages, the infinite, whether with or without specific definition, has been a word to conjure with.

L. E. Hicks.

Augusta, Ga.